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CP asymmetry Relations Between $\bar{B}^0 \to \pi\pi$ And $\bar{B}^0 \to \pi K$ Rates *

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Abstract

We prove that CP violating rate difference $\Delta(\bar{B}^0 \to \pi^+\pi^-) = \Gamma(\bar{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^-\pi^+)$ is related to $\Delta(\bar{B}^0 \to \pi^+ K^-) = \Gamma(\bar{B}^0 \to \pi^+ K^-) - \Gamma(B^0 \to \pi^- K^+)$ in the three generation Standard Model. Neglecting small annihilation diagrams, and in the SU(3) symmetry limit, we show $\Delta(\bar{B}^0 \to \pi^+\pi^-) = -\Delta(\bar{B}^0 \to \pi^+ K^-)$. The SU(3) breaking effects are estimated using factorization approximation, and yield $\Delta(\bar{B}^0 \to \pi^+\pi^-) \approx -(f_\pi/f_K)^2 \Delta(\bar{B}^0 \to \pi^+ K^-)$. Usefulness of this remarkable relation for determining phases in the CKM unitarity triangle is discussed.

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Detection of CP violation and verification of the unitarity triangle of the CKM matrix is a major goal of B factories. Measurement of rate asymmetry in certain channels not only establishes direct CP violation, but can aid in determining some of the angles of the unitarity triangle. In this letter we shall prove a remakable relationship between rate difference $\Delta(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0)) = \Gamma(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0)) - \Gamma(B^0 \to \pi^-\pi^+(\pi^0\pi^0))$ and $\Delta(\bar{B}^0 \to \pi^+K^-(\pi^0\bar{K}^0)) = \Gamma(\bar{B}^0 \to \pi^+K^-(\pi^0\bar{K}^0)) - \Gamma(B^0 \to \pi^-K^+(\pi^0K^0))$. This relationship follows purely from SU(3) symmetry and ignoring annihilation diagrams which can be shown to make negligible contributions. The usefulness of such a relationship lies in that difficult to measure rate difference like $\Delta(\bar{B}^0 \to \pi^+\pi^-)$ can be related to easier measurement of $\Delta(\bar{B}^0 \to \pi^+K^-)$ which is a self-tagging mode. Similarly, it might prove easier to measure $\Delta(\bar{B}^0 \to \pi^0\bar{K}^0)$ than the rate difference of the suppressed mode $\bar{B}^0 \to \pi^0\pi^0$.

In the Standard Model (SM) the effective Hamiltonian for $B \to \pi\pi$ and $B \to \pi K$ decays can be written as follows:

$$
\begin{aligned}
H^q_{eff} = \frac{G_F}{\sqrt{2}}[V_{ub}V^*_{uq}(c_1 O^q_1 + c_2 O^q_2) - \sum_{i=3}^{10}(V_{ub}V^*_{uq}c^u_i \\
+ V_{cb}V^*_{cq}c^c_i + V_{tb}V^*_{tq}c^t_i)O^q_i] + H.C.\ ,
\end{aligned} \tag{1}
$$

where the Wilson coefficients c^f_i are defined at the scale of $\mu \approx m_b$ which have been evaluated to the next-to-leading order in QCD [1], the superscript f indicates the loop contribuiton from f quark, and O^q_i are defined as

$$
\begin{aligned}
&O^q_1 = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha\ , && O^q_2 = \bar{q}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)b\ ,\\
&O^q_3 = \bar{q}\gamma_\mu(1-\gamma_5)b\bar{q}'\gamma_\mu(1-\gamma_5)q'\ , && O^q_4 = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{q}'_\beta\gamma_\mu(1-\gamma_5)q'_\alpha\ ,\\
&O^q_5 = \bar{q}\gamma_\mu(1-\gamma_5)b\bar{q}'\gamma^\mu(1+\gamma_5)q'\ , && O^q_6 = \bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{q}'_\beta\gamma_\mu(1+\gamma_5)q'_\alpha\ ,\\
&O^q_7 = \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)be_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q'\ , && O^q_8 = \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta e_{q'}\bar{q}'_\beta\gamma_\mu(1+\gamma_5)q'_\alpha\ ,\\
&O^q_9 = \frac{3}{2}\bar{q}\gamma_\mu(1-\gamma_5)be_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q'\ , && O^q_{10} = \frac{3}{2}\bar{q}_\alpha\gamma_\mu(1-\gamma_5)b_\beta e_{q'}\bar{q}'_\beta\gamma_\mu(1-\gamma_5)q'_\alpha\ .
\end{aligned} \tag{2}
$$

Here q' is summed over u, d, and s. For $\Delta S = 0$ processes, $q = d$, and for $\Delta S = 1$ processes, $q = s$. O_2, O_1 are the tree level and QCD corrected operators. O_{3-6} are the strong gluon

induced penguin operators, and operators O_{7-10} are due to γ and Z exchange, and "box" diagrams at loop level.

Using the unitarity property of the CKM matrix, we can eliminate the term proportional to $V_{cb}V_{cq}^*$ in the effective Hamiltonian. The B decay amplitude due to the complex effective Hamiltonian displayed above can be paramerized, without loss of generality, as

$$< final\ state|H_{eff}^q|B >= V_{ub}V_{uq}^* T_q + V_{tb}V_{tq}^* P_q \ , \tag{3}$$

where T_q contains the *tree contributions* and *penguin contributions* due to u and c internal quarks, while P_q only contains *penguin contribuitons* from internal c and t quarks.

Since the effective Hamiltonian H_{eff}^d responsible for $\Delta S = 0$ B decays is related to H_{eff}^s for $\Delta S = 1$ B decays by just changing d quark to s quark, one expects certain relations between T_d, P_d and T_s, P_s in the SU(3) limit. Let us consider the two pseudoscalar meson decays of B mesons.

The operators $Q_{1,2}$, O_{3-6}, and O_{7-10} transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, $\bar{3}$, and $\bar{3}_a + \bar{3}_b + 6 + \overline{15}$, respectively. In general, we can write the SU(3) invaraint amplitude for B to two octet pseudoscalar mesons for T_q in the following form [2]

$$\begin{aligned}
T = {} & A^T_{(\bar{3})} B_i H(\bar{3})^i (M_l^k M_k^l) + C^T_{(\bar{3})} B_i M_k^i M_j^k H(\bar{3})^j \\
& + A^T_{(6)} B_i H(6)_k^{ij} M_j^l M_l^k + C^T_{(6)} B_i M_j^i H(6)_l^{jk} M_k^l \\
& + A^T_{(\overline{15})} B_i H(\overline{15})_k^{ij} M_j^l M_l^k + C^T_{(\overline{15})} B_i M_j^i H(\overline{15})_l^{jk} M_k^l \ ,
\end{aligned} \tag{4}$$

where $B_i = (B^-, B^0, B_s^0)$ is a SU(3) triplet, M_{ij} is the SU(3) pseudoscalar octet, and the matrices H represent the transformation properties of the operators O_{1-10}. $H(6)$ is a traceless tensor that is antisymmetric on its upper indices, and $H(\overline{15})$ is also a traceless tensor but is symmetric on its upper indices. For $q = d$, the non-zero entries of the H matrices are given by

$$\begin{gathered}
H(\bar{3})^2 = 1 \ , \ \ H(6)_1^{12} = H(6)_3^{23} = 1 \ , \ \ H(6)_1^{21} = H(6)_3^{32} = -1 \ , \\
H(\overline{15})_1^{12} = H(\overline{15})_1^{21} = 3 \ , \ H(\overline{15})_2^{22} = -2 \ , \ H(\overline{15})_3^{32} = H(\overline{15})_3^{23} = -1 \ .
\end{gathered} \tag{5}$$

For $q = s$, the non-zero entries are

$$
\begin{aligned}
&H(\bar{3})^3 = 1\ ,\ \ H(6)^{13}_1 = H(6)^{32}_2 = 1\ ,\ \ H(6)^{31}_1 = H(6)^{23}_2 = -1\ ,\\
&H(\bar{15})^{13}_1 = H(\bar{15})^{31}_1 = 3\ ,\ H(\bar{15})^{33}_3 = -2\ ,\ H(\bar{15})^{32}_2 = H(\bar{15})^{23}_2 = -1\ .
\end{aligned} \tag{6}
$$

We obtain the amplitudes $T_d(\pi\pi)$, $T_s(\pi K)$ for $\bar{B}^0 \to \pi\pi$, $\bar{B}^0 \to \pi K$ as

$$
\begin{aligned}
T_d(\pi^+\pi^-) &= 2A^T_{(\bar{3})} + C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} + A^T_{(\bar{15})} + 3C^T_{(\bar{15})}\ ,\\
T_d(\pi^0\pi^0) &= \frac{1}{\sqrt{2}}(2A^T_{(\bar{3})} + C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} + A^T_{(\bar{15})} - 5C^T_{(\bar{15})})\ ,\\
T_d(\pi^-\pi^0) &= \frac{8}{\sqrt{2}}C^T_{(\bar{15})}\ ,\\
T_s(\pi^+K^-) &= C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} - A^T_{(\bar{15})} + 3C^T_{(\bar{15})}\ ,\\
T_s(\pi^0\bar{K}^0) &= -\frac{1}{\sqrt{2}}(C^T_{(\bar{3})} - A^T_{(6)} + C^T_{(6)} - A^T_{(\bar{15})} - 5C^T_{(\bar{15})})\ .
\end{aligned} \tag{7}
$$

We also have similar relations for the amplitude P_q. The corresponding amplitude will be denoted by A^P_i and C^P_i. We clearly see the triangle relation (which follows from isospin) holds:

$$
A(\bar{B}^0 \to \pi^0\pi^0) + A(B^- \to \pi^-\pi^0) = \frac{1}{\sqrt{2}}A(\bar{B}^0 \to \pi^+\pi^-)\ . \tag{8}
$$

As also a similar relation for the charge conjugate decay modes.

The amplitudes $A_{(\bar{3}),(6),(\bar{15})}$ all correspond to annihilation contributions. This can be verified because the light quark index in the B meson is contracted with the Hamiltonian. In the factorization approximation, these amplitudes correspond to the matrix element of the form, for example for $\bar{B}^0 \to \pi^+\pi^-$ decay,

$$
M =< 0|\bar{d}\Gamma^1 b|\bar{B}^0 >< \pi^+\pi^-|\bar{q}\Gamma_2 q|0 >\ , \tag{9}
$$

where q can be u or d quarks. If $\Gamma^1 = \gamma_\mu(1-\gamma_5)$, and $\Gamma_2 = \gamma^\mu(1\pm\gamma_5)$, this matrix element is equal to zero due to vector current conservation. The only exception is when the operators are Fierz transformed, one also obtains a contribution of the type, $\Gamma^1 = 1-\gamma_5$, and $\Gamma_2 = 1+\gamma_5$. However, this contribution is suppressed compared with other contributions. In the factorization approximation, for $q = d$, this contribution is given by,

$$M = i f_B m_B^2 \frac{m_\pi^2}{m_u + m_d} \frac{1}{m_b + m_d} F^{\pi\pi}(m_B^2) \ , \tag{10}$$

where we have used, $< 0|\bar{d}(1-\gamma_5)b|\bar{B}^0 >= i f_B m_B^2/(m_b + m_d)$ and $< \pi^+\pi^-|\bar{d}d|0 >= F^{\pi\pi}(q^2) m_\pi^2/(m_u + m_d)$ [3]. Assuming single pole model for the form factor, $F^{\pi\pi}(q^2) = 1/(1 - q^2/m_\sigma^2)$ with $m_\sigma = 700$ MeV, $F^{\pi\pi}(m_B^2) \approx -0.02$. For $\bar{B}^0 \to \pi^+\pi^-$ we find that the annihilation contribution to $P_d(\pi^+\pi^-)$ is only about 4%, and the contribution to $T_d(\pi^+\pi^-)$ is much smaller. To a good approximation all annihilation amplitudes $A_{(\bar{3}),(6),(\bar{15})}$ can be neglected. From now on we will work in this approximation. We obtain:

$$\begin{aligned}
T_{+-} = T_d(\pi^+\pi^-) = T_s(\pi^+K^-) \ , &\quad P_{+-} = P_d(\pi^+\pi^-) = P_s(\pi^+K^-) \ , \\
T_{00} = T_d(\pi^0\pi^0) = -T_s(\pi^0\bar{K}^0) \ , &\quad P_{00} = P_d(\pi^0\pi^0) = -P_s(\pi^0\bar{K}^0) \ ,
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
A(\bar{B}^0 \to \pi^+\pi^-) &= V_{ub}V_{ud}^* T_{+-} + V_{tb}V_{td}^* P_{+-} \ , \\
A(\bar{B}^0 \to \pi^+K^-) &= V_{ub}V_{us}^* T_{+-} + V_{tb}V_{ts}^* P_{+-} \ , \\
A(\bar{B}^0 \to \pi^0\pi^0) &= V_{ub}V_{ud}^* T_{00} + V_{tb}V_{td}^* P_{00} \ , \\
A(\bar{B}^0 \to \pi^0\bar{K}^0) &= -V_{ub}V_{us}^* T_{00} - V_{tb}V_{ts}^* P_{00} \ .
\end{aligned} \tag{12}$$

Analogus relations have been discussed in the context of obtaining information about penguin contributions to B decays and to determine the unitarity triangle of the CKM matrix [4]. These studies suffer from uncertainties in the strong rescattering phases in the amplitudes. Our derivation spell out the precise assumptions that are necessary to obtain the relations. We shall use them to derive relations between the decay rate differences which do not have uncertainties associated with lack of knowledge of the strong rescattering phases. We have

$$\begin{aligned}
\Delta(\bar{B}^0 \to \pi^+\pi^-) &= -4 Im(V_{ub}V_{ud}^* V_{tb}^* V_{td}) Im(T_{+-}P_{+-}^*) \frac{m_B \lambda_{\pi\pi}}{16\pi} \ , \\
\Delta(\bar{B}^0 \to \pi^+K^-) &= -4 Im(V_{ub}V_{us}^* V_{tb}^* V_{ts}) Im(T_{+-}P_{+-}^*) \frac{m_B \lambda_{\pi K}}{16\pi} \ , \\
\Delta(\bar{B}^0 \to \pi^0\pi^0) &= -4 Im(V_{ub}V_{ud}^* V_{tb}^* V_{td}) Im(T_{00}P_{00}^*) \frac{m_B \lambda_{\pi\pi}}{16\pi} \ ,
\end{aligned}$$

$$\Delta(\bar{B}^0 \to \pi^0 \bar{K}^0) = -4Im(V_{ub}V_{us}^* V_{tb}^* V_{ts}) Im(T_{00}P_{00}^*)\frac{m_B \lambda_{\pi K}}{16\pi} , \tag{13}$$

where $\lambda_{ab} = \sqrt{1 - 2(m_a^2 + m_b^2)/m_B^2 + (m_a^2 - m_b^2)^2/m_B^4}$. In the SU(3) symmetry limit, $\lambda_{\pi\pi} = \lambda_{\pi K}$. Due to the unitarity property of the CKM matrix, for three generations of quarks, $Im(V_{ub}V_{us}^* V_{tb}^* V_{ts}) = -Im(V_{ub}V_{ud}^* V_{tb}^* V_{td})$. We then find

$$\begin{aligned} \Delta(\bar{B}^0 \to \pi^+\pi^-) &= -\Delta(\bar{B}^0 \to \pi^+ K^-) , \\ \Delta(\bar{B}^0 \to \pi^0\pi^0) &= -\Delta(\bar{B}^0 \to \pi^0 \bar{K}^0). \end{aligned} \tag{14}$$

These non-trivial equality relations do not depend on the numerical values of the final state rescattering phases. Of course these relations are true only for three generation model. Therefore they also provide tests for the three generation model.

In the real world the SU(3) symmetry is not exact. The relations obtained above will be modified. We estimate the SU(3) symmetry breaking effects by specific calculations in the factorization approximation. In this approxmation, we have

$$\begin{aligned} T_d(\pi^-\pi^+) &= i\frac{G_F}{\sqrt{2}} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)[\xi c_1 + c_2 + \xi c_3^{cu} + c_4^{cu} + \xi c_9^{cu} + c_{10}^{cu} \\ &+ \frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)}(\xi c_5^{cu} + c_6^{cu} + \xi c_7^{cu} + c_8^{cu})] , \\ T_s(\pi^+ K^-) &= i\frac{G_F}{\sqrt{2}} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)[\xi c_1 + c_2 + \xi c_3^{cu} + c_4^{cu} + \xi c_9^{cu} + c_{10}^{cu} \\ &+ \frac{2m_K^2}{(m_b - m_u)(m_u + m_s)}(\xi c_5^{cu} + c_6^{cu} + \xi c_7^{cu} + c_8^{cu})] , \\ T_d(\pi^0\pi^0) &= i\frac{G_F}{\sqrt{2}} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)[-c_1 - \xi c_2 + \xi c_3^{cu} + c_4^{cu} \\ &+ \frac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu}) - \frac{1}{2}(\xi c_9^{cu} + c_{10}^{cu}) \\ &+ \frac{2m_\pi^2}{(m_b - m_d)(2m_d)}(\xi c_5^{cu} + c_6^{cu} - \frac{1}{2}(\xi c_7^{cu} + c_8^{cu}))] , \\ T_s(\pi^0 \bar{K}^0) &= i\frac{G_F}{\sqrt{2}}\{ f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2)[c_1 + \xi c_2 - \frac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu})] \\ &- i f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)[c_3^{cu} + \xi c_4^{cu} - \frac{1}{2}(\xi c_9^{cu} + c_{10}^{cu}) \\ &- \frac{2m_K^2}{(m_b - m_d)(m_d + m_s)}(\xi c_5^{cu} + c_6^{cu} - \frac{1}{2}(\xi c_7^{cu} + c_8^{cu}))]\} , \end{aligned} \tag{15}$$

where $c_i^{cu} = c_i^c - c_i^u$, $c_i^{ct} = c_i^c - c_i^t$, and $\xi = 1/N_c$ with N_c being the number of color. The amplitude $P_{d,s}$ are obtained by setting $c_{1,2} = 0$ and changing c_i^{cu} to c_i^{ct}. We have used the following decompositions for the form factors

$$
\begin{aligned}
&< \pi^+(q)|\bar{d}\gamma_\mu(1-\gamma_5)u|0 >= if_\pi q_\mu \ , < K^+(q)|\bar{d}\gamma_\mu(1-\gamma_5)u|0 >= if_K q_\mu \ , \\
&< \pi^-(k)|\bar{u}\gamma_\mu b|\bar{B}^0(p) >= (k+p)_\mu F_1^{B\pi} + (m_\pi^2 - m_B^2)\frac{q_\mu}{q^2}(F_1^{B\pi}(q^2) - F_0^{B\pi}(q^2)) \ , \\
&< K^-(k)|\bar{u}\gamma_\mu b|\bar{B}^0(p) >= (k+p)_\mu F_1^{BK} + (m_\pi^2 - m_B^2)\frac{q_\mu}{q^2}(F_1^{BK}(q^2) - F_0^{BK}(q^2)) \ .
\end{aligned}
\tag{16}
$$

In the above we have neglected all annihilation contribuitons which are small compared with other contribuitons as discussed earlier.

Using the fact $m_\pi^2/(m_u + m_d) = m_K^2/(m_u + m_s)$, we obtain

$$
\Delta(\bar{B}^0 \to \pi^+\pi^-) = -\frac{(f_\pi F_0^{B\pi}(m_\pi^2))^2}{(f_K F_0^{B\pi}(m_K^2))^2}\frac{\lambda_{\pi\pi}}{\lambda_{\pi K}}\Delta(\bar{B}^0 \to \pi^+ K^-), \tag{17}
$$

It is clear that in the SU(3) symmetry limit, the above relation reduces to eq.(14). Now we need to use the physical masses for π and K. Assuming single pole for the form factor $F_0^{B\pi}(q^2)$, the form factor has the form $F_0^{B\pi}(q^2) = 1/(1 - q^2/m_{0^+}^2)$ with $m_{0^+} = 5.78$ GeV. To a good approximation, we have $(\lambda_{\pi\pi}/\lambda_{\pi K})(F_0^{B\pi}(m_\pi^2)/F_0^{B\pi}(m_K^2))^2 \approx 1$. We finally obtain

$$
\Delta(\bar{B}^0 \to \pi^+\pi^-) \approx -\frac{f_\pi^2}{f_K^2}\Delta(\bar{B}^0 \to \pi^+ K^-). \tag{18}
$$

For $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \pi^0\bar{K}^0$, the correction is more complicated for two reasons: i) in general $f_\pi F_0^{BK}(m_\pi^2)$ is not equal to $f_K F_0^{B\pi}(m_K^2)$, and ii) the u and d quark masses are not equal. These cause the amplitudes $T(P)_d(\pi^0\pi^0)$ and $T(P)_s(\pi^0\bar{K}^0)$ for $\bar{B}^0 \to \pi^0\pi^0$ and $\bar{B}^0 \to \pi^0\bar{K}^0$ to be different not simply by an overall factor as in the case for $\bar{B}^0 \to \pi^+\pi^-$ and $\bar{B}^0 \to \pi^+K^-$. However we estimate that the SU(3) breaking effect is about 30%.

The relations obtained above will provide useful means of measuring a phase angle in the unitarity triangle of the CKM matrix. The angle $\alpha = arc(V_{tb}V_{td}^*/V_{ub}V_{ud}^*)$, can be determined by measuring the time dependent CP asymmetry $a(t)_{+-(00)}$ in $\bar{B}^0(B^0) \to \pi^+\pi^-(\pi^0\pi^0)$ decays [5,6]. The coefficient of the term varying with time as $\sin(\Delta m t)$ is proportional to $Im\lambda_{+-(00)}$ where

$$\lambda_{+-(00)} = (V_{tb}^* V_{td}/V_{tb} V_{td}^*) \frac{A(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0))}{A(B^0 \to \pi^-\pi^+(\pi^0\pi^0))} \; . \tag{19}$$

If penguin contributions are ignored, one finds $A(\bar{B}^0 \to \pi^+\pi^-)/A(B^0 \to \pi^-\pi^+) = V_{ub}V_{ud}^*/V_{ub}^*V_{ud}$, and $\mathrm{Im}\lambda_{+-(00)} = -\sin(2\alpha)$. However, the penguin contributions have been shown to be important [7] and can not be ignored. The relation changes to:

$$\mathrm{Im}\lambda_{+-(00)} = -\frac{|A(B^0 \to \pi^+\pi^-)|}{|A(\bar{B}^0 \to \pi^-\pi^+)|} \sin(2\alpha + \theta_{+-(00)}) \; . \tag{20}$$

A method has been suggested to remove uncertainties due to this change by depermining $\theta_{+-(00)}$ which involves reconstruction of the triangle relation of eq.(8) [6]. This requires precise measurement of rate difference $\Delta(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0))$. It is difficult to measure these rate differences because all the decay modes require tagging. The rate difference $\Delta(\bar{B}^0 \to \pi^+K^-)$, on the other hand, is much easier to measure. Similarly, we can get information for $\Delta(\bar{B}^0 \to \pi^0\pi^0)$ from the measurement of $\Delta(\bar{B}^0 \to \pi^0\bar{K}^0)$. In this case the rate difference $\Delta(\bar{B}^0 \to \pi^0\bar{K}^0)$ is also a difficult quantity to measure because it also needs tagging. However it might be easier to measure compared with $\Delta(\bar{B}^0 \to \pi^0\pi^0)$ since $\bar{B}^0 \to \pi^0\pi^0$ is expected to be highly suppressed.

We would also like to remark that if in the future $\Delta(\bar{B}^0 \to \pi^+K^-(\pi^0\bar{K}^0))$ and $\Delta(\bar{B}^0 \to \pi^+\pi^-(\pi^0\pi^0))$ are all measured precisely, the relations obtained above will provide tests for the three generation model because additional contributions to the decay rates from physics beyond the three generation SM will change these relations.

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